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The trace anomaly of the $(2,0)$ tensor multiplet in background gauge fields

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Abstract: We study the trace anomaly of the $(2,0)$ tensor multiplet in $d=6$ in the presence of a background SO(5) vector field acting as a source for the R-current. Using both a free-field theory calculation and AdS_7/CFT_6 correspondence, we find that only one of the two possible anomaly structures is non-zero and that its coefficient at strong-coupling differs by the well-known overall factor $4N^3$ from the corresponding weak coupling result. We also discuss the relevance of our result to studies of the R-current anomaly in the $(2,0)$ multiplet.

Keywords: AdS-CFT Correspondance, Anomalies in Field and String Theories.

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Contents

1. Some general remarks

The conjectured AdS/CFT correspondence [1] provides a rare tool for studying the strong coupling dynamics of certain gauge theories. Up to now, the correspondence has found its best application in studies of the conjectured duality between type IIB string theory on $AdS_5 \times S^5$ describing the dynamics of N coincident $D3$-branes and the large-N, large-$g_{YM}^2 N$ limit of $\mathcal{N} = 4$ SYM in $d = 4$ with gauge group SU(N). This so-called AdS_5/CFT_4 correspondence is the prototype example for all possible dualities between various compactifications of string/M-theory and gauge field theories.

Of particular importance in establishing the AdS_5/CFT_4 correspondence are studies of the trace and the SO(6) R-current anomalies in $\mathcal{N} = 4$ SYM_4. Such studies have revealed, among others, the field theory interpretation of the parameter N, which in the string theory picture corresponds to the number of the coincident $D3$-branes, as being the dimension of the gauge group. Since the operators in the gauge theory transform under the adjoint representation of an SU(N) gauge group, the conformal anomaly in the large-N strong coupling regime differs from the corresponding weak-coupling value by an overall N^2 factor. Then, as the conformal anomaly is in the same supermultiplet with the R-current anomaly [2], the large-N strong coupling value of the latter also differs by the same overall factor N^2 from the corresponding weak-coupling value [3, 4, 5, 6].

Recently, [7, 8, 9, 10, 11, 14] there has been growing interest in studying the AdS_7/CFT_6 correspondence as another example of the duality between string/M-theory and gauge field theory. Explicitly, this form of the correspondence is conjectured to encode the duality between M-theory compactifications on $AdS_7 \times S^4$ and the maximally supersymmetric $(2,0)$ tensor multiplet in $d=6$. The former theory describes the low energy limit of N coincident $M5$-branes. The latter is a mysterious, strongly coupled six-dimensional CFT without a free coupling parameter. Nevertheless, there also exists in $d=6$ a free $(2,0)$ tensor multiplet which would, presumably, describe the weak-coupling regime of the above mysterious theory. In studying the AdS_7/CFT_6 correspondence one naturally focuses on the conformal and R-symmetry anomalies of the boundary CFT_6 theory. Conformal anomaly studies have revealed a remarkable property [9, 10, 11]: the Weyl-invariant part of the conformal anomaly in the strong-coupling regime of the $(2,0)$ multiplet differs by an overall $4N^3$ factor from the corresponding weak-coupling anomaly, the latter being calculated using free-fields. The field-theoretic interpretation of the overall $4N^3$ factor — yet alone of the N parameter — is far from obvious in this case and it is conceivably related to some as yet unknown realization of gauge invariance in higher dimensions.

As supersymmetry relates the trace and the SO(5) R-current anomalies (the energy momentum and the R-current are in the same supermultiplet), it is important that studies of the trace anomaly are compatible with the well-known result [12, 13] for the the R-current anomaly of the $(2,0)$ theory. In this work we present an explicit calculation of the trace anomaly of the $(2,0)$ tensor multiplet in the presence of a background SO(5) vector field, both using a free-field realization and also using AdS_7/CFT_6 correspondence. In this way our calculation yields respectively the weak- and the strong-coupling results for the trace anomaly. In the next section we briefly review the structure of the trace anomaly in $d=6$ in the presence of background vector fields and discuss its connection to the coefficients of the two- and three-point functions of R-currents. Then we present our calculation. The free-field calculation is done using Seeley-de Witt coefficients while for the strong-coupling calculation we rely on AdS_7/CFT_6 correspondence. We find that only one of the two possible trace anomaly structures is non-zero. We attribute the result to the maximal supersymmetry of the $(2,0)$ tensor multiplet. The strong-coupling result differs from the weak-coupling one by an overall factor $4N^3$. Finally, we discuss the relevance of our result to the well-known results for the R-current anomaly of the $(2,0)$ tensor multiplet in $d=6$.

2. Trace anomaly and R-current correlation functions in the $(2,0)$ tensor multiplet

Let $V_\mu(x) = V_\mu^A(x)T^A$, $\mu = 1,\ldots,d$ be a general conserved current of a d-dimensional CFT. In the case when this coincides with the R-current of the $(2,0)$ multiplet in $d=6$, T^A denote the adjoint generators of SO(5). When the theory is coupled to a background vector field $A_\mu^A(x)$, even in flat spacetime the trace of the energy momentum tensor acquires an (external) anomaly which up to total derivative terms can be written as [6]

$$\langle T_\mu{}^\mu(x)\rangle = \alpha_V\, F_\mu^{A,\nu} F_\nu^{B,\lambda} F_\lambda^{C,\mu} f^{ABC} + \beta_V\, \nabla^\mu F_{\mu\nu}^A\, \nabla^\lambda F_\lambda^{A,\nu}\,, \tag{2.1}$$

where $F_{\mu\nu}^A(x)$ is the standard field strength of $A_\mu^A(x)$. Notice in (2.1) the presence of two different structures in in contrast to the $d=4$ case where only one structure appears. An important property of the parameters α_V, β_V in (2.1) is that they are intimately connected to the parameters appearing in the two- and three-point functions of the R-current $V_\mu^A(x)$. Such a connection follows from the observation that the external trace anomaly is tied to the short distance singularities of renormalized n-point functions. In the case of interest here, assuming a coupling to the background vector field of the form $\int \mathrm{d}^d x\, \sqrt{g}\, g^{\mu\nu} A_\mu^A(x) V_\nu^A(x)$ we can follow [16] and write the general renormalization group equation as

$$\begin{aligned}
&\sum_{k=1}^{\infty}\frac{1}{k!}\int \mathrm{d}^6x_1\sqrt{g}g^{\mu_1\nu_1}\cdots \mathrm{d}^6x_k\sqrt{g}g^{\mu_k\nu_k}\times\\
&\times A_{\mu_1}^{A_1}(x_1)\cdots A_{\mu_k}^{A_k}(x_k)\,\mu\frac{\partial}{\partial\mu}\langle V_{\nu_1}^{A_1}(x_1)\cdots V_{\nu_k}^{A_k}(x_k)\rangle_R = \\
&\qquad = \int \mathrm{d}^6x\sqrt{g}g^{\mu\nu}(x)\langle T_{\mu\nu}(x)\rangle\,.
\end{aligned} \tag{2.2}$$

The subscript R in the first line of (2.2) denotes the renormalized n-point functions which depend on the arbitrary mass parameter μ. Taking suitable functional derivatives of (2.2) with respect to $A_\mu^A(x)$ we can, in principle, connect the parameters which appear in n-point functions of $J_\mu^A(x)$ with the possible terms in the trace anomaly. Then, the importance of (2.2) is based on the fact that in a CFT the two- and three-point functions of conserved currents are determined for general d up to a number of constant parameters. Specifically, for the case of the conserved current $V_\mu^A(x)$ one has [17]

$$\langle V_\mu^A(x)V_\nu^B(y)\rangle = \delta^{AB}\frac{\mathcal{C}_V^{(d)}}{[(x-y)^2]^{(d-1)}}I_{\mu\nu}(x-y)\,,\quad I_{\mu\nu}(x)=\delta_{\mu\nu}-2\frac{x_\mu x_\nu}{x^2}\,, \tag{2.3}$$

$$\begin{aligned}
\langle V_\mu^A(x)V_\nu^B(y)V_\lambda^C(z)\rangle &= \frac{f^{ABC}}{(x-y)^{d-2}(x-z)^{d-2}(y-z)^d}\times\\
&\quad\times I_{\nu\sigma}(x-y)\, I_{\lambda\rho}(x-z)\, I_{\sigma\beta}(X)\, t_{\mu\beta\rho}(X)\,,\\
X_\mu &= \frac{(x-y)_\mu}{(x-y)^2}-\frac{(x-z)_\mu}{(x-z)^2}\,,\\
t_{\mu\nu\lambda}(X) &= \mathcal{A}^{(d)}\frac{X_\mu X_\nu X_\lambda}{X^2}+\mathcal{B}^{(d)}\left(X_\mu\delta_{\nu\lambda}+X_\nu\delta_{\mu\lambda}-X_\lambda\delta_{\mu\nu}\right),
\end{aligned} \tag{2.4}$$

Therefore, had one been able to provide the explicit relation between the parameters $\mathcal{C}_V^{(6)}$, $\mathcal{A}^{(6)}$, $\mathcal{B}^{(6)}$ and α_V, β_V, the calculation of the latter two would reduce to a calculation of the former three. In such a case, one could utilize the well-known AdS_7/CFT_6 calculations for the two- and three-point functions of conserved currents [3] to calculated the trace anomaly.

Using differential renormalization arguments, the parameter β_V was evaluated in terms of $\mathcal{C}_V^{(6)}$ as [6]

$$\beta_V = \frac{\mathcal{C}_V^{(6)}\pi^3}{960}\,. \tag{2.5}$$

The corresponding result for α_V is still missing, however on general grounds one expects it to be a linear combination of $\mathcal{A}^{(6)}$ and $\mathcal{B}^{(6)}$.

The above show that both the weak- and the strong-coupling values of the trace anomaly parameters α_V and β_V for the $(2,0)$ tensor multiplet can be obtain from the corresponding values of $\mathcal{C}_V^{(6)}$, $\mathcal{A}^{(6)}$ and $\mathcal{B}^{(6)}$. The weak-coupling values of the latter three parameters can be found using a free-field realization for the theory as [6]

$$\mathcal{C}_{V,\mathrm{free}}^{(6)} = \frac{5}{\pi^6}\,, \qquad \mathcal{B}_{\mathrm{free}}^{(6)} = \frac{9}{2\pi^9}\,, \qquad \mathcal{A}_{\mathrm{free}}^{(6)} = \frac{3}{\pi^9}\,. \tag{2.6}$$

The strong-coupling values can be found using AdS_7/CFT_6 correspondence which requires the consideration of maximal supergravity on AdS_7.[1] The relevant part of the supergravity lagrangian is

$$\mathcal{L} = -\frac{1}{4g_{SG_7}^2}\int \mathrm{d}^7\hat{x}\,\sqrt{g}\,F_{ij}^A(\hat{x})F^{A,ij}(\hat{x})\,, \qquad i,j=0,\ldots,6\,, \tag{2.7}$$

from where one obtains [3, 4]

$$\mathcal{C}_V^{(6)} = \frac{120}{\pi^3 g_{SG_7}^2}\,, \qquad \mathcal{A}^{(6)} = \frac{72}{\pi^6 g_{SG_7}^2}\,, \qquad \mathcal{B}^{(6)} = \frac{108}{\pi^6 g_{SG_7}^2}\,. \tag{2.8}$$

The important remaining piece of information is the value of $1/g_{SG_7}^2$. This can be read from the general results for $d+1 = 4,5,7$, given in [6]. One considers the equations of motion of gauged supergravity in 4, 5 and 7 dimensions correspondingly in the presence of non trivial scalar fields in the coset space $\mathrm{SL}(n,\mathbb{R})/\,\mathrm{SO}(n)$. The $\mathrm{SO}(n)$ group corresponds to the R-symmetry group of the boundary CFT_d. The result is

$$\frac{1}{g_{SG_{d+1}}^2} = \frac{n(n-2)}{4d(d-1)}\frac{1}{2\kappa_{d+1}^2} = \frac{1}{2\kappa_{d+1}^2}\frac{2}{(d-2)^2}\,, \tag{2.9}$$

where we have used the relation $n = 4\frac{d-1}{d-2}$ [18]. Then, from (2.6) and (2.8) we obtain[2]

$$\frac{\mathcal{C}_V^{(6)}}{\mathcal{C}_{V,\mathrm{free}}^{(6)}} = \frac{\mathcal{B}^{(6)}}{\mathcal{B}_{\mathrm{free}}^{(6)}} = \frac{\mathcal{A}^{(6)}}{\mathcal{A}_{\mathrm{free}}^{(6)}} = 4N^3\,. \tag{2.10}$$

[1] We consider the euclidean version of AdS_{d+1} space where $\mathrm{d}\hat{x}^i\mathrm{d}\hat{x}_i = \frac{1}{x_0^2}(\mathrm{d}x_0\mathrm{d}x_0 + \mathrm{d}x^\mu\mathrm{d}x^\mu)$, with $\mu = 1,\ldots,d$. and $\hat{x}_i = (x_0, x_\mu)$. The boundary of this space is isomorphic to S^d since it consists of $\mathbb{R}^d$ at $x_0 = 0$ and a single point at $x_0 = \infty$.

[2] It is important to point out that the general result (2.9) is compatible with all known calculations of two- and three-point functions in $d = 3,4,6$ (see e.g. [15]). In particular, for $d = 3$ we obtain $\mathcal{C}_V^{(3)}/\mathcal{C}_{V,\mathrm{free}}^{(3)} = \mathcal{B}^{(3)}/\mathcal{B}_{\mathrm{free}}^{(3)} = \mathcal{A}^{(3)}/\mathcal{A}_{\mathrm{free}}^{(3)} = N^{3/2}4\sqrt{2}/3\pi$ which shows that the ratio between the strong- and weak-coupling values for the two- and three-point functions in $d = 3$ is $N^{3/2}4\sqrt{2}/3\pi$. This irrational overall factor coincides with the one found in [8] in studies of the two- and three-point functions of the energy momentum tensor.

Then, from (2.5) and (2.10) we obtain

$$\beta_V = 4N^3 \beta_{V,\text{free}} = \frac{N^3}{192\pi^3}\,, \tag{2.11}$$

Our result shows that the strong-coupling value of the trace anomaly parameter β_V differs from its weak-coupling value by an overall $4N^3$ factor.

3. The free-field result

In the absence of a result such as (2.5) for the trace anomaly parameter α_V, we have to rely on some other method for calculating the trace anomaly (2.1) both in the weak- and also in the strong-coupling regimes of the $(2,0)$ tensor multiplet. In each case, agreement with the result (2.11) would be a strong test for our calculation.

The weak-coupling values of both coefficients α_V and β_V can be calculated by the method of Seeley-De Witt coefficients using a free-field realization of the $(2,0)$ tensor multiplet. Following [10] we can evaluate the anomaly in $d = 6$ from the Seeley-De Witt coefficient b_6 for the general second order Laplace operator

$$\Delta = -\nabla^2 - E\,, \tag{3.1}$$

where ∇_μ is a covariant derivative with normal bundle connection $[\nabla_\mu, \nabla_\nu] = F_{\mu\nu}$ and E is a matrix endomorphism. The general formula for b_6 is (see [10] and references therein)

$$\begin{aligned}
b_6 &= \frac{1}{(4\pi)^3}\left(\alpha_6 + \frac{1}{6}\alpha_2^3 + \alpha_2\alpha_4\right), && (3.2)\\
\alpha_2 &= E\,, && (3.3)\\
\alpha_4 &= \frac{1}{6}\nabla^2 E + \frac{1}{12}F_{\mu\nu}^2\,, && (3.4)\\
\alpha_6 &= \frac{2}{6!}\Big[8(\nabla_\alpha F_{\mu\nu})^2 + 2(\nabla^\alpha F_{\alpha\mu})^2 + 12F_{\alpha\beta}\nabla^2 F^{\alpha\beta} - \\
&\qquad - 12F_\alpha{}^\mu F_\mu{}^\beta F_\beta{}^\alpha + 6\nabla^4 E + 30(\nabla_\alpha E)^2\Big]. && (3.5)
\end{aligned}$$

We have to evaluate b_6 for fermions and bosons taking into account the following identities

$$\begin{aligned}
\nabla_\alpha F_{\mu\nu}\nabla^\alpha F^{\mu\nu} &= 2\nabla^\mu F_{\mu\alpha}\nabla^\nu F_\nu{}^\alpha - 2F_\alpha{}^\mu F_\mu{}^\beta F_\beta{}^\alpha + \nabla_\alpha J_1^\alpha\,,\\
J_1^\alpha &= F_{\nu\mu}\nabla^\mu F^{\nu\alpha} - F_\nu{}^\alpha\nabla_\mu F^{\nu\mu}\,.
\end{aligned} \tag{3.6}$$

The last total derivative term in (3.6) is not important here as it can be cancelled by adding local counterterms [20], therefore we can drop it in our calculation.

For scalar bosons we have

$$
\begin{aligned}
F_{\mu\nu} = F^A_{\mu\nu} T^A \,, \qquad E = 0\,, \qquad \mathrm{Tr}\,(F_{\mu\nu}F^{\mu\nu}) &= -C_\varphi F^A_{\mu\nu}F^{A,\mu\nu}\,, \\
\mathrm{Tr}\left(F_\alpha{}^\mu F_\mu{}^\beta F_\beta{}^\alpha\right) &= -\frac{1}{2} C_\varphi f^{ABC} F^{A,\mu}_\alpha F^{B,\beta}_\mu F^{C,\alpha}_\beta \,, \\
b^s_6 &= \frac{C_\varphi \left(\nabla^\alpha F^A_{\alpha\mu}\right)^2}{(4\pi)^3 60} + \frac{C_\varphi f^{ABC} F^{A,\mu}_\alpha F^{B,\beta}_\mu F^{C,\alpha}_\beta}{(4\pi)^3 180}\,.
\end{aligned} \tag{3.7}
$$

The corresponding spinor contribution is

$$
\begin{aligned}
F_{\mu\nu} = F^A_{\mu\nu}T^A \mathbf{I}_\psi \,, \qquad E = \frac{1}{2} F^A_{\mu\nu} T^A \gamma^{\mu\nu}\,, \qquad \mathrm{Tr}\,(F_{\mu\nu}F^{\mu\nu}) &= -C_\psi (\mathrm{Tr}\,\mathbf{I}_\psi) F^A_{\mu\nu}F^{A,\mu\nu}\,, \\
\mathrm{Tr}\left(F_\alpha{}^\mu F_\mu{}^\beta F_\beta{}^\alpha\right) &= -\frac{1}{2} C_\psi (\mathrm{Tr}\,\mathbf{I}_\psi) f^{ABC} F^{A,\mu}_\alpha F^{B,\beta}_\mu F^{C,\alpha}_\beta \,, \\
b^f_6 &= -\frac{C_\psi (\mathrm{Tr}\,\mathbf{I}_\psi) \left(\nabla^\alpha F^A_{\alpha\mu}\right)^2}{(4\pi)^3 15} + \frac{C_\psi (\mathrm{Tr}\,\mathbf{I}_\psi) f^{ABC} F^{A,\mu}_\alpha F^{B,\beta}_\mu F^{C,\alpha}_\beta}{(4\pi)^3 180}\,.
\end{aligned} \tag{3.8}
$$

From (3.7) and (3.8) we can calculate the free-field result for the total trace anomaly of the $d = 6$, $(2,0)$ tensor multiplet in the the presence of external vector fields and up to total derivatives terms we obtain

$$
\begin{aligned}
\langle T_\mu{}^\mu(x)\rangle &= b^s_6(x) - b^f_6(x) \\
&= \frac{\left(\nabla^\alpha F^A_{\alpha\mu}\right)^2}{(4\pi)^3 15}\left(\frac{C_\varphi}{4} + C_\psi (\mathrm{Tr}\,\mathbf{I}_\psi)\right) + \frac{f^{ABC} F^{A,\mu}_\alpha F^{B,\beta}_\mu F^{C,\alpha}_\beta}{(4\pi)^3 180}\Big(C_\varphi - C_\psi (\mathrm{Tr}\,\mathbf{I}_\psi)\Big) \\
&= \frac{\mathcal{C}^{(6)}_{V,\mathrm{free}}\pi^3}{960}\left(\nabla^\alpha F^A_{\alpha\mu}\right)^2 + 0\cdot f^{ABC} F^{A,\mu}_\alpha F^{B,\beta}_\mu F^{C,\alpha}_\beta\,.
\end{aligned} \tag{3.9}
$$

The get from the second line to the third in (3.9) we used the general expression for $\mathcal{C}^{(6)}_{V,\mathrm{free}}$ [17] and also the following selection rule obtained in [6]

$$
C_\varphi = C_\psi (\mathrm{Tr}\,\mathbf{I}_\psi)\,, \tag{3.10}
$$

for the free-field realization of the $(2,0)$ tensor multiplet. The value of β_V read off from (3.9) coincides up to an overall $4N^3$ factor with (2.11), which is a consistency test for our calculation. We also obtain $\alpha_V = 0$ as a direct result of the selection rule (3.10).

4. The strong-coupling result from AdS_7/CFT_6 correspondence

To calculate the anomaly coefficients in the strong coupling limit we use the AdS action (2.7) and follow the methods developed in [9, 21]. For that, we have to consider the on-shell dependence of (2.7) on the boundary value of the gauge field and then extract the logarithmic divergence. This is achieved by solving the equations of motion for the Yang-Mills field $A_i(\hat{x}) \equiv A_i^A T^A = (A_0(x_0, x_\mu), A_\mu(x_0, x_\mu))$ in AdS. These equations are significantly simplified by choosing to work on the gauge $A_0 = 0$, which is a natural gauge condition preserving gauge invariance on the boundary. In this gauge the equations of motions following from (2.7) are

$$\begin{aligned} &\hat{\nabla}_j F^{ji} \equiv \frac{1}{\sqrt{g}}\partial_j\left(\sqrt{g}F^{ji}\right) + \left[A_j, F^{ji}\right] = 0\,, \\ &\hat{\nabla}_\mu F_{\mu x} = 0\,, \qquad x_0^{d+1}\partial_0\left(x_0^{-d-1+4}F_{x\mu}\right) + x_0^4 \hat{\nabla}_\nu F_{\nu\mu} = 0\,. \end{aligned} \tag{4.1}$$

Then, following [21] (see also [22]), we expand the vector fields around the conformal boundary in a power series as

$$A_\mu(x_0, x_\mu) = \sum_{k=0}^{\infty} x_0^k A_\mu^{(k)}(x_\mu)\,, \qquad F_{0\mu} = \partial_0 A_\mu = \sum_{k=1}^{\infty} k x_0^{k-1} A_\mu^{(k)}(x_\mu)\,, \tag{4.2}$$

$$F_{\mu\nu}(x_0, x_\mu) = \sum_{k=0}^{\infty} x_0^k F_{\mu\nu}^{(k)}(x_\mu) = \sum_{k=0}^{\infty} x_0^k \left(\partial_\mu A_\nu^{(k)} - \partial_\mu A_\nu^{(k)} + \sum_{l=0}^{k}\left[A_\mu^{(l)}, A_\nu^{(k-l)}\right]\right). \tag{4.3}$$

The boundary value of the vector field is $A_\mu^{(0)}(x)$, consequently in order to be able to extract the logarithmic singularity of the action (2.7) we have in the following to introduce a suitable IR regulator in the x_0-integration. Here we just mention that a simple expansion such as (4.2) implemented with a suitable IR regularization of the x_0-integration can be easily shown to give the correct conformal anomaly for massless scalars in any dimension. Substituting (4.2) and (4.3) into (4.1) we obtain

$$\begin{aligned} \sum_{k=1}^{\infty}(2+k-d)k x_0^{k+2} A_\mu^{(k)} &= -\sum_{k=1}^{\infty} x_0^{k+3}\nabla_\nu F_{\nu\mu}^{(k-1)} - \sum_{k=1}^{\infty} x_0^{k+4}\sum_{l=0}^{k}\left[A_\nu^{(l)}, F_{\nu\mu}^{(k-l)}\right], \\ \sum_{k=1}^{\infty} k x_0^{k-1}\nabla_\mu A_\mu^{(k)} &= -\sum_{k=2}^{\infty} x_0^{k-1}\sum_{l=1}^{k-1} l\left[A_\mu^{(k-l)}, A_\mu^{(l)}\right], \end{aligned} \tag{4.4}$$

where $\nabla_\mu = \partial_\mu + [A_\mu^{(0)}, \ldots]$. The equations (4.4) can be recursively solved for all k. Here we consider only the relevant to us cases $k = 1, 2, \ldots$ when we obtain the following set of solutions

$$\begin{aligned} A_\mu^{(1)} &= 0\,, \\ A_\mu^{(2)} &= -\frac{1}{2(4-d)}\nabla_\nu F_{\nu\mu}^{(0)} \overset{(d=6)}{=} \frac{1}{4}\nabla_\nu F_{\nu\mu}(0)\,, \nabla_\mu A_\mu^{(2)} = 0\,, \\ A_\mu^{(3)} &= 0\,. \end{aligned} \tag{4.5}$$

Plugging in (4.5) into (2.7) we can extract the logarithmic term as

$$-\frac{1}{4g_{SG_7}^2}\int_\epsilon^\infty \frac{dx_0}{x_0}\int d^6x\left(8A_\mu^{(2)}A_\mu^{(2)} + 4F_{\mu\nu}^{(0)}\nabla_\mu A_\nu^{(2)}\right) = \ln\epsilon \frac{1}{8g_{SG_7}^2}\int d^6x\left(\nabla^\mu F_{\mu\nu}^{(0)}\right)^2. \tag{4.6}$$

In (4.6) we used the standard procedure to obtain the regularized generating functional for the boundary CFT_6 by considering the x_0-coordinate as an IR regulator in the bulk ($x_0 \in (\infty, \epsilon), \epsilon \to 0$) which corresponds to an UV regulator in the boundary $\mu = 1/\epsilon$ dropping at the same time all bulk UV divergences. Then, from (2.8) we finally get by virtue of (2.10)

$$\langle T_\mu{}^\mu(x) \rangle_{AdS_7} = 0 \cdot F_\mu^{A,\nu} F_\nu^{B,\lambda} F_\lambda^{C,\mu} f^{ABC} + \frac{\mathcal{C}_V^{(6)} \pi^3}{960} \left(\nabla^\mu F_{\mu\nu}^A \right)^2 . \tag{4.7}$$

5. Discussion

Our results (3.9) and (4.7) for the trace anomaly of the $(2,0)$ multiplet in the presence of external gauge fields were obtained in the absence of external gravity. They are characterized both by the manifestation of the overall $4N^3$ as one goes from the weak-coupling (free-fields) to the strong-coupling regime and also by the vanishing of the one of the two possible structures (namely $\alpha_V = 0$). The latter fact can be attributed, as seen from (3.9), to the selection rule (3.10) which in turn may be viewed as a manifestation of maximal supersymmetry both in the supergravity and also in the boundary CFT_d.

Moreover, the results above are connected to the well-known results for the R-current anomaly [2]. The structure of the supersymmetry algebra in $d = 6$ is, however, quite involved and an explicit relation between the trace and the R-symmetry anomalies has not appeared in the literature as yet. Nevertheless, a separate discussion of the known results for the trace and R-current anomalies of the $(2,0)$ multiplet might be useful.

In the absence of an external gravitational background the R-current anomaly is given by the following 8-form

$$I_8^{\text{free}}(F) = \frac{1}{3 \cdot 2^4} \left[p_2(F) + \frac{1}{4} p_1(F)^2 \right] , \tag{5.1}$$

$$p_1(F) = \frac{1}{2} \operatorname{tr} \tilde{F}^2 , \qquad p_2(F) = -\frac{1}{4} \left[\operatorname{tr} \tilde{F}^4 - \frac{1}{2} \operatorname{tr} \tilde{F}^2 \wedge \operatorname{tr} \tilde{F}^2 \right] , \qquad \tilde{F} = \frac{i}{2\pi} F . \tag{5.2}$$

Eq. (5.1) gives the anomaly $\mathcal{I}_6$ in the six-dimensional theory via the descent equations $d(\delta \mathcal{I}_6) = \delta I_7$, $I_8 = \delta I_7$. Notice that, at first sight, the descent formalism seems to give two possible linearly independent structures for the six-dimensional anomaly. This has to be compared with our trace anomaly results (3.9) and (4.7) which involve only one structure.

The R-symmetry anomaly of the strongly-coupled $(2,0)$ multiplet has been also evaluated requiring the cancellation of the total anomaly of N $M5$-branes when one takes into account the inflow anomaly [12]. The result in the absence of external gravity is [13]

$$I_8^{(2,0)}(F) = \frac{1}{3\cdot 2^4}\left[(2N^3 - N)p_2(F) + \frac{N}{4}p_1(F)^2\right]. \tag{5.3}$$

Now, if AdS_7/CFT_6 correspondence is valid, one should be able to recover the large-N limit of the latter result by considering the maximally supersymmetric $N = 2$ gauged SUGRA in $d = 7$. Namely, one should find a result which lifted to 8-dimensions should read

$$I_8^{(AdS/CFT)}(F) = \frac{2N^3}{3\cdot 2^4}p_2(F). \tag{5.4}$$

One then observes that (5.1) and (5.4) seem to imply that the R-symmetry anomaly has different structure in the weak (free-fields) and the strong-coupling regimes. This in turn implies some kind of renormalization of the R-symmetry anomaly as one goes from the weak to the strong coupling regimes and it is reminiscent to the corresponding result for the trace anomaly of the $(2,0)$ multiplet in an external gravitation background in which at least the Euler density term seems to be also renormalized [10, 11]. Moreover, by virtue of supersymmetry (5.1) and (5.4) seem to indicate that the corresponding result for the trace anomaly in the presence of external vector fields, but in the absence of external gravity, would also be renormalized as one goes from the weak to the strong-coupling regimes. Such a conclusion appears to be, at first sight, incompatible with our result (3.9) and (4.7) i.e. that the structure of the trace anomaly is the same in both the weak and the strong coupling regimes. It is conceivable that a better understanding of supersymmetry in $d = 6$ might resolve this apparent puzzle.

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